PONDICHERRY UNIVERSITY-5643037

B.Tech. DEGREE EXAMINATION, APRIL 2015

Third Semester-Electrical and Electronics Engineering

ELECTROMAGNETIC THEORY (2013–2014 Onwards)

Time: Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions. All questions carry equal marks.

1. State Coulomb's law.
   Ans 1. For answer refer: **Page 32, Topic 1.7** till formula

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}^2}.$$

2. State Gauss's law.
   Ans 2. For answer refer: **Page 37, Topic 1.9** only greyed box.

3. Define dielectric strength.
   Ans 3. For answer refer: **Page 61, Topic 2.3** only greyed box.

4. What is Laplace's Equation?

   Ans 4. $$\left.\begin{aligned} &\nabla^2 V = 0 \\ &\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \end{aligned}\right\} \text{This is called Laplace's Equation.}$$

5. State Ampere's circuital law.
   Ans 5. For answer refer: **Page 112, Topic 3.6**.

6. Define magnetic flux density.
   Ans 6. For answer refer: **Page 105, Topic 3.2** only.

7. State Faraday's law.
   Ans 7. For answer refer: **Page 154, Topic 4.2** till $V = N\left(\frac{d\phi}{dt}\right)$.

8. Define inductance of a solenoid.

   Ans 8. $L = \left(\frac{\mu N^2 A}{l}\right)$.

9. Define poynting vector.
   Ans 9. For answer refer: **Page 203, Topic 5.8 & Page 204** first greyed box.

10. What is displacement current?
    Ans 10. For answer refer: **Page 68, Topic 2.6.3** till $I_D = \left(\frac{\varepsilon A}{d}\right)\left(\frac{dV}{dt}\right)$.

PART B – (5 × 11 = 55 marks)

Answer ALL questions, ONE from each unit. All question carry equal marks.

UNIT I

11. (a) Obtain expression for electric field intensity due to line charge, surface charge and volume charge distributions. (6)
Ans 11. (a) For answer refer: **Page 37, Topic 1.9**, grey box.

For answer refer: **Page 39, Topic 1.9.3** till **Topic 1.9.5** in Page 41 " $E = \left(\dfrac{\rho_v r_1^3}{3\varepsilon_0 r_2^2}\right)$ ".

[As it is for only 6 marks, derivation for a finite line charge, finite surface charge and volume charge is very long and not required]

11. (b) Derive Maxwell's first equation. (5)
Ans 11. (b) For answer refer: **Page 163** Maxwell's Eq I only.

Or

12. (a) State and prove divergence theorem. (6)
Ans 12. (a) For answer refer: **Page 30, Topic 1.6.3** fully.

12. (b) Obtain expression for electric potential due to a line charge. (5)
Ans 12. (b) For answer refer: **Page 56, Topic 2.1.2**.

UNIT II

13. (a) Obtain mathematical expression for polarization in dielectrics. (6)
Ans 13. (a) For answer refer: **Topic 2.5** in page 63 only. [(ie) till Dielectric Polarization]

13. (b) Derive expression of field relation for time varying electromagnetic fields continuity of current equation in point form. (5)
Ans 13. (b) For answer refer: **Topic 2.12.2** in page 87 till Eq (I) in page 88.

Or

14. (a) Derive Poisson's equation from the fundamental postulates of electrostatics. (6)
Ans 14. (a) For answer refer: **Topic 2.7** in page 70.

14. (b) Determine whether or not the following potential fields satisfy the Laplace's equation:
(i) $V = x^2 - y^2 + z^2$
(ii) $V = r\cos\phi + Z$
(iii) $V = \cos\theta + \phi$. (5)

Ans 14. (b) (i) $V = x^2 - y^2 + z^2$

$$\frac{\partial V}{\partial x} = 2x; \quad \frac{\partial V}{\partial y} = -2y; \quad \frac{\partial V}{\partial z} = 2z$$

$$\frac{\partial^2 V}{\partial x^2} = 2; \qquad \frac{\partial^2 V}{\partial y^2} = -2; \qquad \frac{\partial^2 V}{\partial z^2} = 2$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 2 - 2 + 2 = 2$$

Eq (i) Does not satisfy Laplace's Equation.

Ans 14. (b) (ii) $V = r\cos\phi + \text{z}$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$$

(i) $\frac{\partial V}{\partial z} = 0 + 1; \qquad \frac{\partial^2 V}{\partial z^2} = 0$ (1)

(ii) $\frac{\partial V}{\partial r} = \cos\phi; \quad r\frac{\partial V}{\partial r} = r\cos\phi; \quad \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = \cos\phi$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = \left(\frac{\cos\phi}{r}\right); \tag{2}$$

(iii) $\frac{\partial V}{\partial \phi} = r[-\sin\phi]; \quad \frac{\partial^2 V}{\partial \phi^2} = -r\cos\phi;$

$$\frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) = \frac{-\cos\phi}{r}; \tag{3}$$

Adding (1) + (2) + (3), we have,

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \left(\frac{\partial^2 V}{\partial z^2}\right) = 0.$$

So Eq (ii) satisfies the Laplace's Equation.

Ans 14. (b) (iii) $V = \cos\theta + \phi.$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 V}{\partial \phi^2}\right).$$

$$\frac{\partial V}{\partial \phi} = 1; \qquad \frac{\partial^2 V}{\partial \phi^2} = 0; \tag{1}$$

$$\left(\frac{\partial V}{\partial \theta}\right) = -\sin\theta; \qquad \sin\theta\frac{\partial V}{\partial \theta} = -\sin^2\theta$$

$$\sin\theta\left(\frac{\partial V}{\partial \theta}\right) = \frac{\cos 2\theta - 1}{2}$$

$$\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = \frac{1}{\not{2}}\left[-\sin 2\theta\cdot(\not{2}) - 0\right]$$
$$= -\sin 2\theta = -2\sin\theta\cdot\cos\theta$$

$$\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)\right] = \frac{-2\cos\theta}{r^2} \quad (2)$$

$$\frac{\partial V}{\partial r} = 0; \qquad r^2\frac{\partial V}{\partial r} = 0; \qquad \frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0;$$

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)\right] = 0 \quad (3)$$

$$\nabla^2 V = (1) + (2) + (3) = \frac{-2\cos\theta}{r^2} \neq 0.$$

So Eq (iii) Does not satisfy the Laplace's Equation.

UNIT III

15. (a) State and prove Stokes's Theorem. (6)

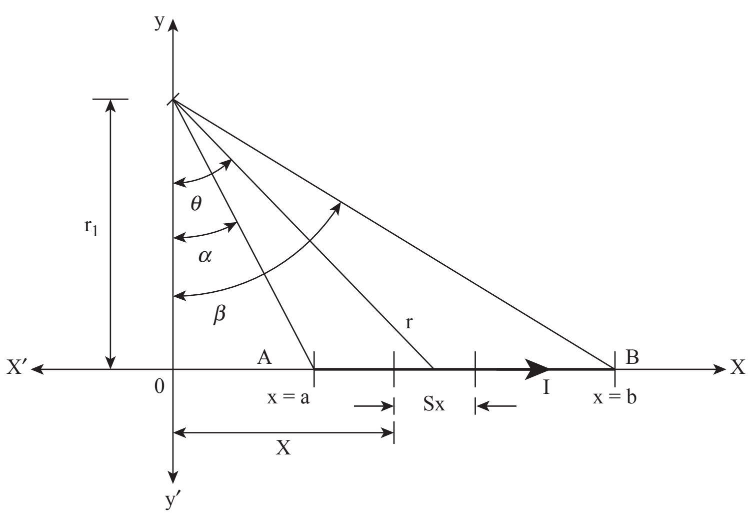
Ans 15. (a) For answer refer: **Topic 1.6.1 in page 28** fully till page 30 first 2 lines.

15. (b) Obtain an expression for magnetic vector potential in the region surrounding an infinitely long straight filamentary current I. (5)

Ans 15. (b) For answer refer: **Page 126, Topic 3.9.2**.

Magnetic vector potential is defined as

$$\vec{A} = \int_L \frac{\mu_0 I(dl)}{4\pi R}$$



Let's find the magnetic vector potential, $\vec{A}$ for a conductor of finite length and then let's proceed to find for a conductor of infinite length.

$$\vec{A} = \left(\frac{\mu_0 I}{4\pi}\right) \int_{x=a}^{x=b} \frac{dx}{r} \qquad \text{(I)}$$

$$r^2 = x^2 + r_1^2$$

$$r = \sqrt{x^2 + r_1^2}$$

$$\vec{A} = \left(\frac{\mu_0 I}{4\pi}\right) \int_{x=a}^{x=b} \frac{dx}{\sqrt{x^2 + r_1^2}}$$

$$= \left(\frac{\mu_0 I}{4\pi}\right) ln\left[\left(x + \sqrt{x^2 + r_1^2}\right)\right]_{x=a}^{x=b}$$

$$\vec{A} = \left(\frac{\mu_0 I}{4\pi}\right) ln\left[\frac{b + \sqrt{b^2 + r_1^2}}{a + \sqrt{a^2 + r_1^2}}\right]$$

For an Infinite length conductor, the magnetic vector potential is given by

$$\vec{A} = \left(\frac{\mu_0 I}{2\pi}\right) \cdot ln\, r.$$

Or

16. Derive the expression for a curl, applying Ampere's circuital law to an incremental surface element. (11)

Ans 16. For answer refer: **Page 26, Topic 1.5.3** till $\nabla \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$

For answer refer: **Page 163**, Maxwell's Eq (I), fully.

UNIT IV

17. (a) Determine the force per metre length between two long parallel wires A and B separated by 5 cm in air and carrying current of 400 amps in the same direction. (6)

Ans 17. (a) For answer refer: **Page 146**, **Q(1)**; only value of current I = 400 A is changed.

17. (b) Obtain expression for coefficient of coupling between two inductor in terms of self inductance and mutual inductance. (5)

Ans 17. (b) Self Inductance

Current passing through a conductor produces flux. If changing current flows in a conductor, it produces changing flux, which in turn induces an emf in the conductor.
The emf so induced is proportional to the rate of change of current.

$$V \alpha \left(\frac{di}{dt}\right); \qquad V = L\left(\frac{di}{dt}\right) \qquad (1)$$

If the coil has "N" no. of turns, the emf induced is given by

$$V = N\left(\frac{d\phi}{dt}\right) \tag{2}$$

In a small interval of time Δt, we have

$$V = L\left(\frac{\Delta i}{\Delta t}\right) = N\left(\frac{\Delta\phi}{\Delta t}\right) \tag{3}$$

If the permeability is constant, then

$$Li = N\phi;\ \ L = \left(\frac{N\phi}{i}\right) \tag{4}$$

Mutual Inductance:

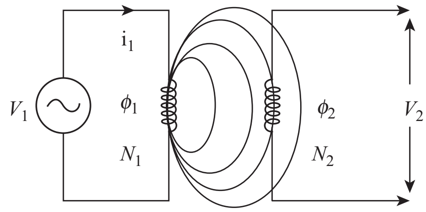


Fig. 3.25 Flux linking from primary to secondary coil.

The AC current in coil 1, having N_1 turns produces flux ϕ_1.
A part of the flux ϕ_{12} links coil 2 having N_2 turns.
Emf in coil 2 is given by,

$$v_2 = N_2\left(\frac{d\phi_{12}}{dt}\right) \tag{1}$$

Since v_2 is induced by changing current i_1 in coil 1, we have,

$$v_2 = M\left(\frac{di_1}{dt}\right) \tag{2}$$

where, M = Mutual inductance

$$M\left(\frac{di_1}{dt}\right) = N_2\left(\frac{d\phi_{12}}{dt}\right) \tag{3}$$

For a small period Δt, we have

$$M\left(\frac{\Delta i_1}{\Delta t}\right) = N_2\left(\frac{\Delta\phi_{12}}{\Delta t}\right) \tag{4}$$

Assuming permeability as constant, we have,

$$Mi_1 = N_2\phi_{12}$$

$$M = \left(\frac{N_2\phi_{12}}{i_1}\right) \quad (5)$$

If current i_2 flowing in coil 2 produces flux ϕ_2, a part of which ϕ_{21} link coil N_1, then

$$v_1 = M\left(\frac{di_2}{dt}\right)$$

$$v_1 = N\left(\frac{d\phi_{21}}{dt}\right)$$

$$\Rightarrow M = \left(\frac{N_1\phi_{21}}{i_2}\right) \quad (6)$$

Coefficient of Coupling:
The fraction of the total flux produced by one coil linking 2nd coil is called the coefficient of coupling K.

$$K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2} \qquad \text{K is always } \le 1$$

$$\phi_{12} = K \cdot \phi_1 \qquad \phi_{21} = K \cdot \phi_2$$

Multiplying eq. (5) and (6), we have

$$M^2 = \left(\frac{N_2\phi_{12}}{i_1}\right)\left(\frac{N_1 \cdot \phi_{21}}{i_2}\right) \qquad M^2 = \left(\frac{N_2 \cdot K \cdot \phi_1}{i_1}\right)\cdot\left(\frac{N_1 K \cdot \phi_2}{i_2}\right)$$

$$M = \left(\frac{N_1 \cdot \phi_1}{i_1}\right)\cdot\left(\frac{N_2 \cdot \phi_2}{i_2}\right)K^2 \qquad M^2 = L_1 \cdot L_2 \cdot K^2$$

$$M = K \cdot \sqrt{L_1 L_2} \qquad \Rightarrow M = K\sqrt{L_1 \cdot L_2} \qquad \textbf{(A)}$$

Or

18. (a) Derive expression for inductance of toroid. (6)
Ans 18. (a) For answer refer: **Page 123, Topic 3.7.4** fully.
For answer refer: Page 134, Topic 3.13.2 fully.

18. (b) State the boundary condition for Normal component. (5)
Ans 18. (b) For answer refer: Page 132, Topic 3.12 till $\mu_1 H_{1n} = \mu_2 H_{2n}$. (2)

"The normal component of B is continuous at the boundary. But the normal component of H is discontinuous at the boundary."

UNIT V

19. Obtain expression for electromagnetic wave equations in phasor form. (11)
Ans 19. For answer refer: **Page 176, Topic 5.3** till Eq. (I) in page 177.

For answer refer: **Page 179, Topic 5.3.3** till Eq. (4) in same page.

Or

20. Define and derive skin depth. Calculate the skin depth for a medium with Conductivity 100 mho/m, relative permeability 2, relative permeability 3 at 50 Hz. 1 MHz and 1 GHz. (11)
Ans 20. For answer refer: **Page 188, Topic 5.6** till figure 5.12.

$$\sigma = 100;\ \mu_r = 2;\ f = 50$$
$$\mu_r = 3;\ f = 1\times 10^6 \text{ Hz}$$
$$f = 1\times 10^9 \text{ Hz}$$

$$\left.\begin{matrix}\text{Skin depth}\\ \delta\end{matrix}\right\} = \frac{1}{\sqrt{(\pi f \mu \sigma)}}$$

Case (i); $\sigma = 100;\ \mu_r = 2;\ f = 50;\ 1\times 10^6;\ 1\times 10^9$ Hz

$$\delta_1 = \frac{1}{\sqrt{\pi \times 50 \times 2 \times \mu_0 \times 100}} = 5.03 \text{ m}$$

$$\delta_2 = \frac{1}{\sqrt{\pi \times 1\times 10^6 \times 2 \times \mu_0 \times 100}} = 35.58\times 10^{-3} \text{ m}$$

$$\delta_3 = \frac{1}{\sqrt{\pi \times 1\times 10^9 \times 2 \times \mu_0 \times 100}} = 1.125\times 10^{-3} \text{ m}$$

Case (ii); $\sigma = 100;\ \mu_r = 3;\ f = 50;\ \ f = 1\times 10^6;\ f = 1\times 10^9$ Hz

$$\delta_1 = \frac{1}{\sqrt{\pi \times 50 \times 3 \times \mu_0 \times 100}} = 4.11 \text{ m}$$

$$\delta_2 = \frac{1}{\sqrt{\pi \times 1\times 10^6 \times 3 \times \mu_0 \times 100}} = 29.05\times 10^{-3} \text{ m}$$

$$\delta_3 = \frac{1}{\sqrt{\pi \times 1\times 10^9 \times 3 \times \mu_0 \times 100}} = 918.88\times 10^{-6} \text{ m}$$