Chapter Zero
Introduction to Circuit Theory

0.1 Introduction

All elements in earth are made up of molecules. Molecules are inturn formed by two or more atoms. Atoms are structured in such a way that positively charged nucleus is in the center, surrounded by equal number of negatively charged electrons circulating it. Atom as a whole is neutral as number of protons equals the number of electrons. Structure of Hydrogen, Aluminium and Copper atom is given below.

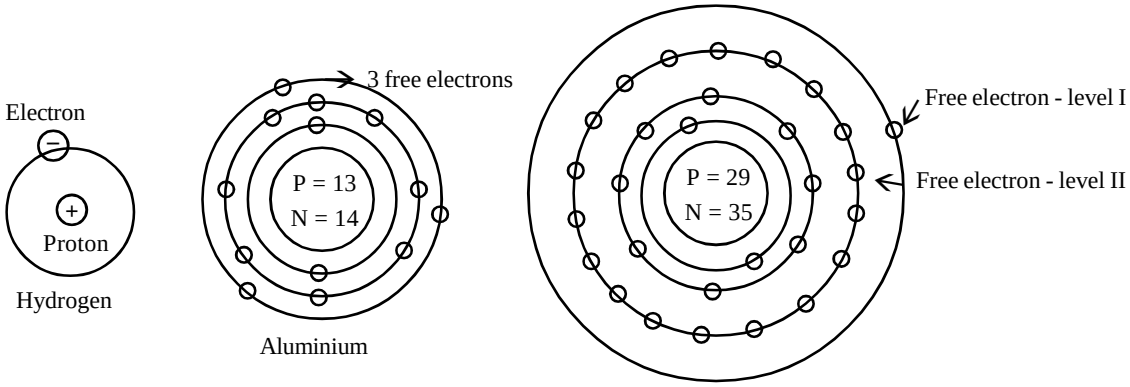

Fig 0.1

In case of insulators, the electrons are tightly bound to the positively charged nucleus. In case of conductors, the electrons in the outer bands are loosely bound to the nuclei. The loosely bound electrons are called free electrons. These free electrons readily move across the conductor, when an electric field or voltage is applied across the conductor.

Let's now study the concept of field, electric field, voltage, voltage source, current and finally circuit theory.

0.2 Field, Electric field, Voltage, Voltage Source, Current and Circuit Theory.

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Field is mathematically defined as a function that specifies a particular quantity to every point within a region.

Field is said to be present in the region, if a particular condition prevails in the region. In other words, if a force or influence is effective or felt in a region, it signifies the presence of the field. The

force or influence is felt by the respective physical entity. For example, if a mass 'M', comes under the influence of the force, it suggests the existence of the gravitational field at that point. If an electron or proton placed at a particular point experiences a force, it suggests the existence of the electric field (or electromagnetic field) at that point.

Let the force experienced by the unit positive charge at a point be 5 N/C, then the electric field intensity "E" at that point is 5 N/C or 5 V/m.

Def : *Electric field or Electric field intensity "E" at a point is defined as the force experienced by a unit positive charge placed at that point.*

$$E = \left(\frac{F}{Q}\right)$$

Where, E = Electric field intensity
F = Force on charge
Q = Magnitude of charge

or the above equation can be written as, $F = Q.E$

Like charges repel and unlike charges attract. So the negative charge at A will be pulled towards D, while positive charge at B will be pushed away from B.

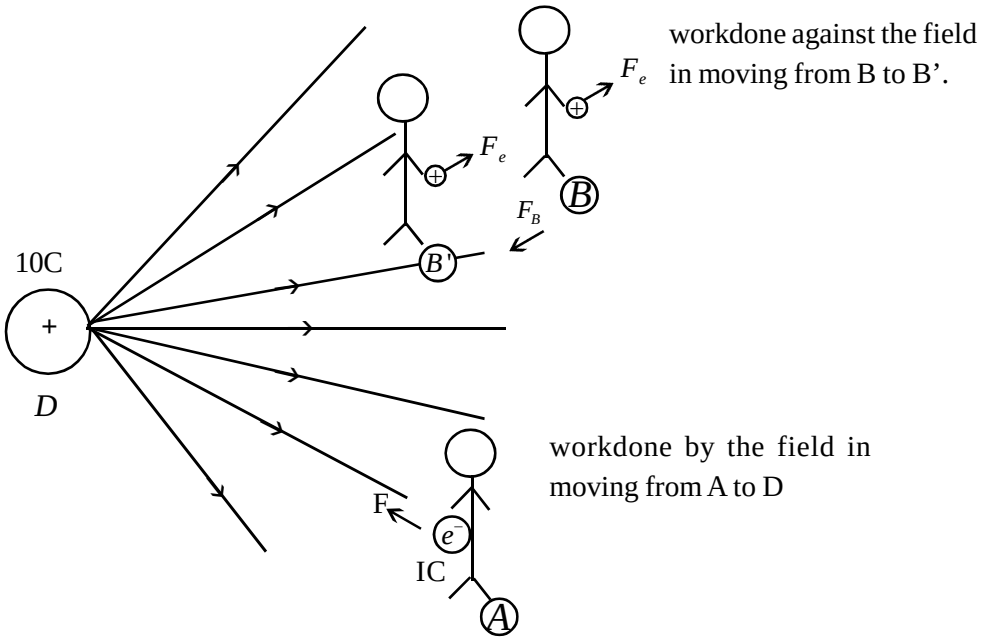


Fig 0.2 Workdone by person carrying positive and negative charges.

Consider a person at point B moving to point B', by applying force $F_{B'}$ against the electrical force of repulsion F_e ($F_B > F_e$). Generally, work done is given by

Workdone, $W = F \bullet d$ = *force* x *distance* = $Fd\cos\theta$ *[dot product]*

As we move towards B', there is an increasing repulsion force and hence we must apply more increasing force to move towards B'. By applying integration formula, we have

$$W = \int_{r_1}^{r_2} F \bullet dl = \int_{r_1}^{r_2} QEdl = Q\int_{r_1}^{r_2} Edl \qquad [\because F = QE]$$

$$W = (1)\int_{r_1}^{r_2} Edl = \int_{r_1}^{r_2} E \cdot dl \qquad [\because q = 1C]$$

Since work is done against the force, we have negative sign.

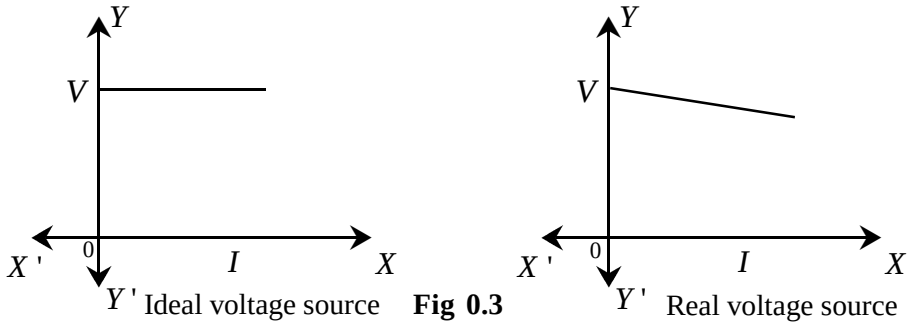
$$W_{BB'} = V_{BB'} = -\int_{B}^{B'} E \cdot dl$$

Potential difference (or voltage difference $V_{BB'}$) is defined as the work done in moving a unit positive charge from one point to another in an electric field. Absolute potential at a point is defined as the work done in moving a unit positive charge from infinity to the point under consideration.

Voltage can only be measured between 2 points by using voltmeter. Voltage measured at a single point will always be zero, as work done at a point is always zero.

Voltage source is one which maintains fixed voltage across its 2 terminals, irrespective of the load conditions.

Real world voltage sources are batteries and generators. The voltage - current characteristics [VI characteristics] of voltage source is given below :



Ideal voltage source **Fig 0.3** Real voltage source

Voltage source is also called as Electromotive Force or EMF. Electromotive force or EMF is the one which gives motive force or the force of motion to the electrons. When the voltage source or EMF is connected by wire (conductor) across the resistor, it does the work of moving the electrons around it. It does so by repelling the electrons at one terminal and attracting the electron at another terminal. In short, voltage sources establishes the flow of electrons across the conductor.

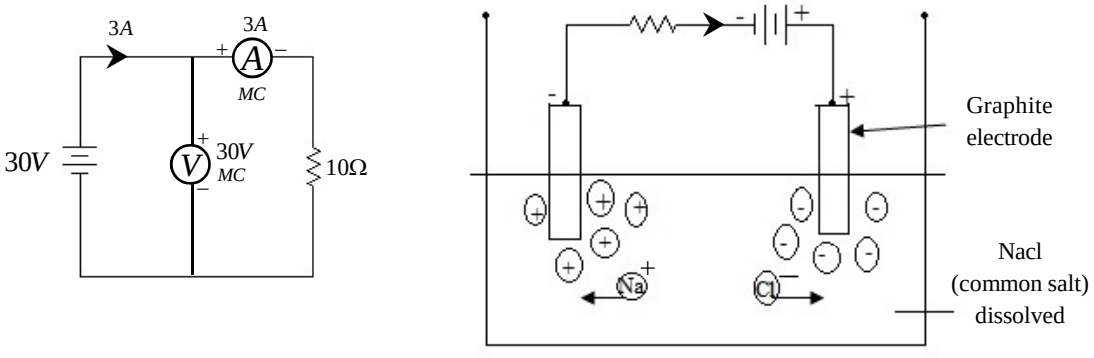


Fig 0.4 Voltage source sets up the flow of current.

In conductor, it is the electrons which flows in the conductor, we assume the direction of current as direction of flow of positive ions (anions). The flow of positively charged particles in a conductor is fictitious and not real.

Only in the case of electrolyte, there is movement of positive ions and negative ions. Consider common salt - Sodium Chloride (NaCl) dissolved in water. It gets split into positively charged Sodium Na^+ ions and negatively charged Chloride Cl^- ions. When subjected to a voltage source through a graphite electrode, the negatively charged chloride ions moves towards anode (+) and the positively charged Sodium Ions moves towards cathode (-).

0.3 Circuit Theory

Circuit is a roughly circular line, route or movement that starts and ends at the same place. In circuit theory, we basically study the flow of current, which starts and ends at the EMF or voltage source. Once the voltage source establishes the flow of current, we need to measure the rate of flow of charge.

Current is defined as the rate of flow of charge. Current is measured at a particular point by using Ammeter.

$$i = \left(\frac{dQ}{dt} \right) \text{Amperes.}$$

We take only the direction of flow of current, which is from positive to negative terminal of voltage source, and totally ignore the flow of electrons henceforth.

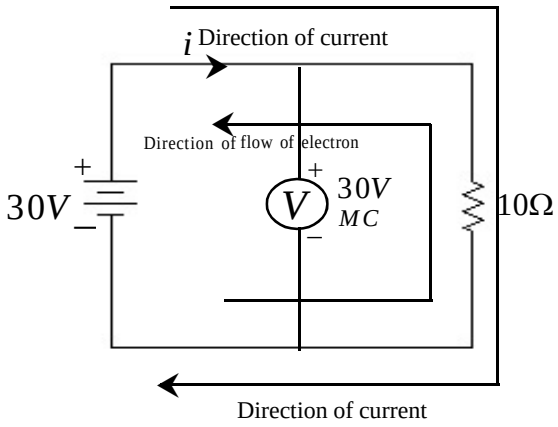


Fig 0.5

0.4 Analogy of Electrical System to Hydraulic System.

Let's consider only DC voltage sources. (For a DC voltage source, inductance appears as short circuit and capacitance appears as open circuit, it has only resistance). We will discuss the AC circuits in detail later.

Just like electric pressure [(ie) voltage V], establishes a current I against the electrical resistance. The hydraulic pressure (P), establishes a water flow at a rate of "q" in the pipe against the hydraulic resistance (R) offered by the pressure valve and the inner rim of the pipe.

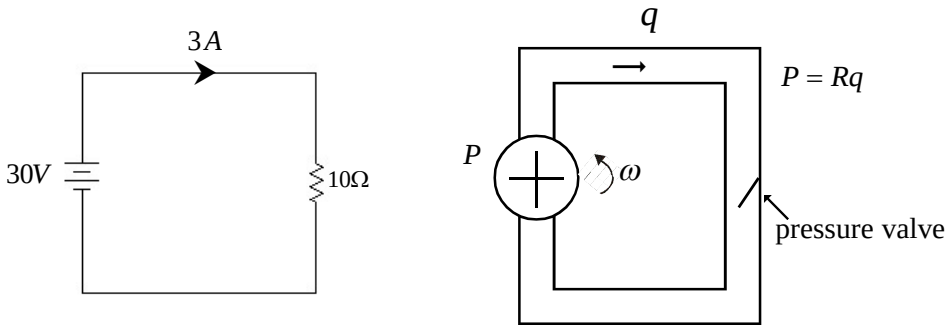


Fig 0.6 Analogy between Electrical and Hydraulic System

Analogy between the parameters of the 2 systems are as follows.

Electrical system	*Hydraulic system*
V = IR	P = Rq
V = Voltage	P = Pressure
I = Current	q = Flow rate
R = Electrical resistance	R = Hydraulic resistance

For an analogy of inductor, capacitor and resistor for electrical system with respect to mechanical and hydraulic system, refer books on control system.

In an electrical circuit, for a DC voltage source, Power is defined as

$$\text{Power = Voltage x Current;} \qquad \Rightarrow P = VI$$

DC voltage is scalar, whereas electric field is vector. AC voltage is a phasor and its addition and subtraction necessarily follows the vector laws.

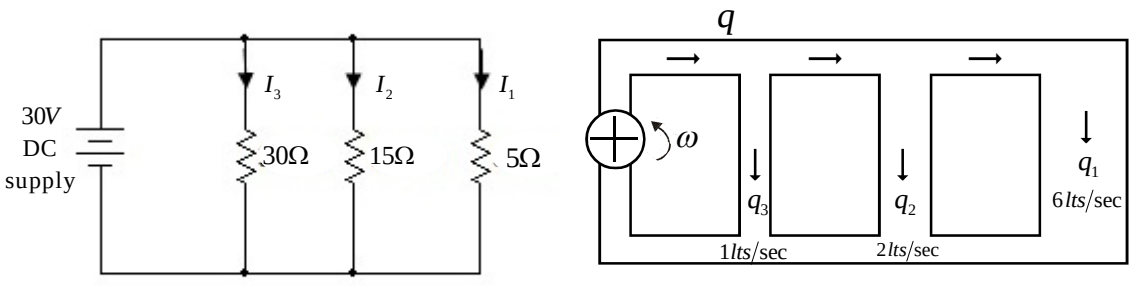


Fig 0.7 Analogy between Electrical and Hydraulic System

Consider the 3 branch parallel resistive network. The current flowing through the branch is given by

$$I_1 = \frac{30}{5} = 6A; \qquad I_2 = \frac{30}{15} = 2A; \qquad I_3 = \frac{30}{30} = 1A$$

Resistance (R) is the opposition offered by the material to the flow of current.

Electrical Resistance is given by $R = \left(\frac{\rho l}{A}\right)$, where

ρ = resistivity of the material; ℓ = length of material; A = Area of cross section.

If resistance is more, then opposition is more, which implies less current flows through the branch.

Greater the area of cross section, lesser the resistance and hence higher the flow of current (or water in analogy to hydraulics).

Consider the hydraulic system above. It is obvious that more water flows through the wider pipe and less water flows through thin pipe. This is because thin pipes has small cross section and high resistance, while wider pipe has larger cross section and less resistance.

0.5 Field Theory and Circuit Theory Comparison

Sl.No.	Circuit Theory	Field Theory
1.	Analysis is restricted within a conductor	Analysis is restricted to 2 or 3 dimensional space.
2.	Independent and dependent parameters V & I are studied in circuit theory	Electric field E and magnetic field H, it variation and propagation in space are studied in field theory.
3.	Ohm's law and Kirchhoff's law forms the corner stone of analysis in circuit theory	Maxwell's equation forms the corner stone of analysis.
4.	Parameters of the medium are not involved. Only parameters like R, L, C of a conductor is used.	Parameters of the medium in 2 dimensional or 3 dimensional space, like permitivity, ε and permeability, μ are involved in the analysis.
5.	Applicable only for a portion of the frequency range. Eg. 10 to 1000 Hz.	Applicable over a wide range of frequency in terms of MHz, GHz etc.
6.	Low power is involved in circuit theory. If high power comes into picture, it becomes power system analysis.	Relatively high power is involved.
7.	Lumped components and parameters are involved in analysis.	Distributed components and parameters are involved in analysis. Eg. 50 Ω / meter. 20 H / km etc.
8.	Z, Y and H parameters are used	S parameter is used
9.	Two dimensional analysis and it is simple to understand	3 - Dimensional analysis and needs visualization ability.