# Anna University-Question Paper Code-77124

## B.E /B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

## Third Semester, Electrical and Electronics Engineering

## EE 6302–ELECTROMAGNETIC THEORY

## (Regulation 2013)

Time: Three hours | Maximum : 100 marks

**PART A — (10 × 2 = 20 marks)**

**Answer ALL questions. All questions carry equal marks.**

1. Given $A = 4a_x + 6a_y - 2a_z$ and $B = -2a_x + 4a_y + 8a_z$ show that the vectors are orthogonal.
**Ans 1.** Similar problem solved in page 48, Q1.

$$\vec{A} = 4a_x + 6a_y - 2a_z$$
$$\vec{B} = -2a_x + 4a_y + 8a_z$$

$$A \bullet B = -8 + 24 - 16 = 0$$

$$\Rightarrow A \bullet B = |A|\,|B|\cos\theta = 0; \quad \Rightarrow \theta = 90°$$

Hence $\vec{A}$ and $\vec{B}$ are orthogonal to each other.

2. Express in matrix form the unit vector transformation from the rectangular to cylindrical co-ordinate system.
**Ans 2.** Unit vector transformation from rectangular to cylindrical system. **[formula in page 12]**

$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

3. What is the Practical significance of Lorentz's forces?
**Ans 3.** Lorentz's force deflects the electron beam in TV (CRT TV) and CRT monitor to get the picture.

4. Find the electric field intensity in free space if D = 30 $a_x$ C/$m^3$.

**Ans 4.** $D = \varepsilon_0 E;\; E = \left(\dfrac{D}{\varepsilon_0}\right)$. **[formula in page 40]**

$$E = \frac{30\hat{a}_x}{\varepsilon_0} = 3.38 \times 10^{12} \text{ V/m}$$

5. Find the force of interaction between two charges 4 × $10^{-8}$ and 6 × $10^{-5}$ spaced 10cm apart in kerosene $(\varepsilon_r = 2.0)$.

**Ans 5.** $Q_1 = 4 \times 10^{-8}$ C; $\qquad Q_2 = 6 \times 10^{-5}$ C

$$d = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$$
$$\varepsilon_r = 2$$

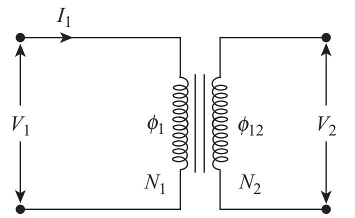
$$F = \frac{Q_1 \cdot Q_2}{4\pi\varepsilon_0 d^2} = \frac{4 \times 10^{-8} \times 6 \times 10^{-5}}{4\pi\, \varepsilon_0\, \varepsilon_r (10 \times 10^{-2})^2}$$
$$F = 1.08\, N.$$

6. Find the maximum torque on an 100 turns rectangular coil of 0.2 m by 0.3 m, carrying current of 2A in the field of flux density 5 web/m$^2$.
**Ans 6.** $T_{max}$ = BIA **[formula in page 124]**

For "N" turns,

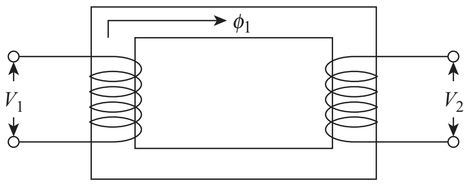
$$T_{\max} = NBIA = 100 \times 5 \times 2 \times 0.2 \times 0.3$$
$$T_{\max} = 60 \text{ N.m.}$$

7. Define mutual inductance and self inductance.




Let a voltage source $V_1$ set up a flux $\phi_1$ per turn in the primary of the transformer taking current $I_1$.

Then self Inductance is given by $L = \left(\frac{N\phi_1}{I_1}\right)$



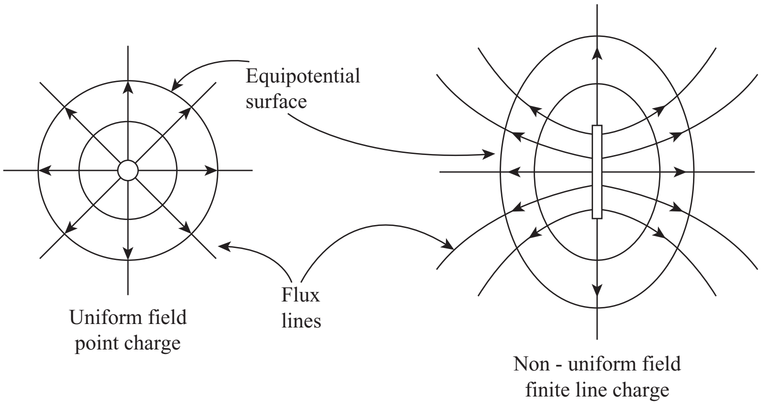

Let the flux linking the secondary be $\phi_{12}$ per turn, then mutual inductance is given by,

$$M = \left(\frac{N_2\phi_{12}}{I_1}\right).$$

8. Distinguish between transformer emf and motional emf.
**Ans 8.** For answer refer: **Topic 4.4.1 and Topic 4.2.2**.

| Transformer EMF | Motional EMF |
|---|---|
| 1. Here flux is alternating and there is no movement. | 1. Here flux is constant and the conductor is moving. |
| 2. Eg. Transformer. | 2. Eg. DC generator. |

9. Compare the equi-potential plots of uniform and non-uniform fields.
**Ans 9.** For answer refer: **Page 60, Topic 2.2.3**.




10. What is the wavelength and frequency of a wave propagation in free space when $\beta = 2$?
**Ans 10. Formula in page 186**

Velocity of wave, $v = \left(\frac{\omega}{\beta}\right)$

$$v = 3\times10^8 \text{ m/sec} ; \beta = 2$$

$$\Rightarrow \omega = v\cdot\beta = 3\times10^8 \times 2 = 6\times10^8 \text{ rad/sec}$$

$$\omega = 2\pi f = 6\times10^8$$

$$\Rightarrow f = \frac{6\times10^8}{2\pi} = 95.49\times10^6 \text{ Hz}$$

$$\lambda f = v$$

$\Rightarrow$ wavelength, $\lambda = \left(\frac{v}{f}\right) = 3.14$ m.

**PART B – (5 × 16 = 80 marks)**

11. (a) (i) State and prove Gauss's Law. (8)
**Ans 11. (a) (i)** For answer refer: **page 37, Topic 1.9** for statement only; refer **page 38 Topic 1.9.1** for proof.

11. (a) (ii) Obtain an expression for electric field intensity due to a uniformly charged line of length "ℓ". (8)

**Ans 11. (a) (ii)** For answer refer: **Page 32, Topic 1.8-**Grey box for definition and Topic 1.8.1 fully.

Or

11. (b) (i) Show that over the closed surface of a sphere of radius b, ds = 0. (6)

**Ans 11 (b) (i)** Question seems incomplete.

11. (b) (ii) Show that the vector $E = (6xy + z^3)a_x + (3x^2 - z)a_y + (3xz^2 - y)a_z$ is irrotational and find its scalar potential. (10)

**Ans 11. (b) (ii)** $\vec{E} = \left(6xy + z^3\right)\hat{a}_x + \left(3x^2 - z\right)\hat{a}_y + \left(3xz^2 - y\right)\hat{a}_z$

for a vector field to be irrotational, $\nabla \times \vec{E} = 0.$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ (6xy + z^3) & (3x^2 - z) & (3xz^2 - y) \end{vmatrix}$$ **[formula in page 26]**

$$= \hat{a}_x \left[\frac{\partial}{\partial y}(3xz^2 - y) - \frac{\partial}{\partial z}(3x^2 - z)\right] - \hat{a}_y \left[\frac{\partial}{\partial x}(3xz^2 - y) - \frac{\partial}{\partial y}(6xy + z^3)\right] + \hat{a}_y \left[\frac{\partial}{\partial x}(3x^2 - z) - \frac{\partial}{\partial y}(6xy + z^3)\right]$$

$$= \hat{a}_x \left[(0-1) - (0-1)\right] - \hat{a}_y \left[(3z^2 - 0) - (0 + 3z^2)\right] + \hat{a}_z \left[(6x - 0) - (6x + 0)\right]$$

$$\nabla \times \vec{E} = 0\,\hat{a}_x + 0\,\hat{a}_y + 0\,\hat{a}_z = 0.$$

Since "Curl E" is zero, the vector field is irrotational
Relation between "V" and "E" is given by

$$\nabla V = -E = -\left[\left(6xy + z^3\right)\hat{a}_x + \left(3x^2 - z\right)\hat{a}_y + \left(3xz^2 - y\right)\hat{a}_z\right].$$

(ie) $$\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z = -\left[\left(6xy + z^3\right)\hat{a}_x + \left(3x^2 - z\right)\hat{a}_y + \left(3z^2 - y\right)\hat{a}_z\right].$$

$$\frac{\partial V}{\partial x} = +\left(6xy + z^3\right)$$

$$\partial V = +\left(6xy + z^3\right)dx$$

$$V_x = +\left[\int 6y\,x dx + \int z^3\,dx\right]$$

$$V_x = +\left[6y\left(\frac{x^2}{2}\right) + z^3 x\right] = +\left[3x^2 y + xz^3\right]$$

$$\frac{\partial V}{\partial y} = +\left(3x^2 - z\right)$$

$$dV_y = \left(3x^2 - z\right)dy$$

$$V_y = \int 3x^2 dy - \int z dy = 3x^2 y - zy$$

$$\frac{\partial V}{\partial z} = +\left(3xz^2 - y\right)$$

$$dV_z = \left(3xz^2 - y\right)\cdot dz$$

$$V_z = \int 3xz^2 dz - \int y\,dz = 3x\left(\frac{z^3}{3}\right) - yz$$

$$V_z = xz^3 - yz$$

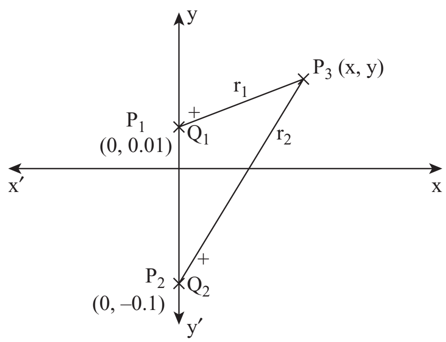
$$V = V_x + V_y + V_z = \left(3x^2 y + xz^3\right) + \left(3x^2 y - zy\right) + \left(xz^3 - yz\right)$$

$$V = 6x^2 y + 2xz^3 - 2yz = 2\left[3x^2 y + xz^3 - yz\right].$$

12. (a) (i) A positive point charge $100 \times 10^{-12}$ C is located in air at $x = 0$, 0.01m and another such charge at x = 0, y = –0.1 m.. What is the magnitude and direction of E? (6)

**Ans 12. (a) (i)** $Q_1 = 100 \times 10^{-12}$ C $P_1(0, 0.01)$

$Q_2 = 100 \times 10^{-12}$ C $P_2(0, -0.1)$




Let's assume a random point "$P_3$" $(x, y)$

$$\vec{E}_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1^2}\hat{a}_{r_1}$$

$$\vec{E}_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2^2}\hat{a}_{r_2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0\left[(x-0)^2+(y-0.01)^2\right]}\hat{a}_{r_1} + \frac{Q_2}{4\pi\varepsilon_0\left[(x-0)^2+(y+0.1)^2\right]}\hat{a}_{r_2}$$

As $Q_1 = Q_2 = 100\times10^{-12}\,\text{C}$, we have,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\hat{a}_{r_1}}{x^2+(y-0.01)^2} + \frac{\hat{a}_{r_2}}{x^2+(y+0.1)^2}\right]$$

$$= \frac{100\times10^{-12}}{4\pi\varepsilon_0}\left[\frac{\hat{a}_{r_1}}{x^2+(y-0.01)^2} + \frac{\hat{a}_{r_2}}{x^2+(y+0.1)^2}\right]$$

$$\vec{E} = 900\times10^{-3}\left[\frac{\hat{a}_{r_1}}{x^2+(y-0.01)^2} + \frac{\hat{a}_{r_2}}{x^2+(y+0.1)^2}\right].$$

12. (a) (ii) Obtain an expression for the capacitance of a parallel plate capacitor with two dielectrics of relative permittivity $\varepsilon_1$ and $\varepsilon_2$ respectively interposed between the plates. (10)
**Ans 12. (a) (ii)** For answer refer: **page 72, Topic 2.8.3 fully**.

Or

12. (b) Explain the polarization and thus obtain electric field intensity and potential of a dipole. (16)
**Ans 12. (b)** For answer refer: **page 63** and take all 3 statements from grey boxes with diagram-Fig 2.18.

For "E" and "V" of Dipole refer **page 64, Topic 2.5.1 and 2.5.2** till page 66 end.

13. (a) (i) Obtain an expression for magnetic flux density and magnetic field intensity at any point due to a finite length conductor. (8)
**Ans 13. (a) (i)** For answer refer: **page 116 "Topic 3.7.1"** till 117 page end.

13. (a) (ii) State and prove magnetic boundary conditions. (8)
**Ans 13. (a) (ii)** For answer refer: **page 132 Topic 3.12** fully.

Or

13. (b) Obtain an expression for inductance and torque on a long solenoid coil. (16)
**Ans 13. (b)** For answer refer: **page 122 Topic 3.7.3** till Eq.

$\vec{B} = \frac{\mu_0 IN}{2L}\left[\cos\alpha_1 + \cos\alpha_2\right]$ in page 123 and **Topic 3.13.1 in page 134**.

14. (a) Derive the set of Maxwell's equations with solutions in integral form from fundamental laws for a good conductor. (8)
**Ans 14. (a)** For answer refer: **Page 163, Topic 4.5.2** till page 166 Eq. IV.

Or

14. (b). (i) Explain the relation between field theory and circuit theory and thus obtain an expression for ohm's law. (8)

**Ans 14. (b) (i)** For answer refer: **page 167, Topic 4.5.3**. In Electromagnetic theory, point form of ohm's law is given by

$$J = \sigma E$$

$$\frac{I}{A} = \frac{E}{\rho}\left(\because \ \sigma = \frac{1}{\rho}\right) \qquad \Rightarrow \left(\frac{\rho}{A}\right) I = E$$

where, $\sigma$ = conductivity; $\rho$ = resistivity
E = Electric field Intensity; J = Current density
R = Resistance; I = Conduction current.

For a conductor of length "ℓ"; we have,

$$\left(\frac{\rho \ell}{A}\right) \cdot I = E\ell$$

$$RI = V \qquad \left[\because R = \left(\frac{\rho\ell}{A}\right)\right]$$

(ie) V = IR (Ohm's law)

14. (b) (ii) Compare and explain in detail conduction and displacement currents. (8)

**Ans 14. (b) (ii)** For answer refer: **page 68 Topic 2.6.2 and 2.6.3**.

15. (a) (i) State Poynting theorem and thus obtain an expression for instantaneous power density vectors associated with electromagnetic fields. (12)

**Ans 15. (a) (i)** For answer refer: **Page 203, Topic 5.8** till 205 page ending.

15. (a) (ii) A Plane wave travelling in air is normally incident in a block of paraffin with $\varepsilon_r = 2.2$. Find the reflection coefficient. (4)

**Ans 15. (a) (ii)** Reflection coefficient, $\Gamma = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)$

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_1}}; \qquad \eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}}$$

Medium 1 is Air Medium 2 is paraffin

Since conductivity is not given in both the cases, let's assume $\sigma_1 = 0$; and $\sigma_2 = 0$

$$\Rightarrow \eta_1 = \sqrt{\left(\frac{\mu_1}{\varepsilon_1}\right)} \qquad \eta_2 = \sqrt{\left(\frac{\mu_2}{\varepsilon_2}\right)}$$

$$\eta_1 = \sqrt{\left(\frac{\mu_0 \ \mu_r}{\varepsilon_0 \ \varepsilon_r}\right)} \qquad \eta_2 = \sqrt{\left(\frac{\mu_0 \ \mu_r}{\varepsilon_0 \ \varepsilon_r}\right)}$$

For air $\mu_r = 1$; $\varepsilon_r = 1$ $\qquad \varepsilon_r = 2.3$; $\mu_r = 1$ (assumed)

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.73 \qquad \eta_2 = \sqrt{\frac{\mu_0}{(2.3)\varepsilon_0}} = 248.40$$

$$\Gamma = \frac{248.40 - 376.73}{248.40 + 376.73} = -0.205.$$

15. (b) Obtain an expression for electromagnetic wave propagation in lossy dielectrics.
**Ans 15 (b)** For answer refer: **page 176, Topic 5.3** till wave Eq (I) and page 181 Topic 5.4 till

$$E = C_1 e^{-\beta x} + C_2 e^{j\beta x}.$$