PONDICHERRY UNIVERSITY-5623028

B.Tech. DEGREE EXAMINATION, APRIL 2015.

Third Semester-Electronics and Communication Engineering

ENGINEERING ELECTROMAGNETICS (2013-2014 onwards)

Time: Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions. All questions carry equal marks.

1. What is a Gradient?
   Ans 1. For answer refer: **Page 23**, last greyed box.

2. State Coulomb's Law.
   Ans 2. For answer refer: **Page 32, topic 1.7**, fig 1.32 & formulae.

3. Define absolute electric potential.
   Ans 3. For answer refer: **Page 55, topic 2.1**.

4. Give formula for conduction current density and displacement current density.
   Ans 4. For answer refer: **Page 68, topic 2.6.2; page 69, Topic 2.6.3**

$$J_C = \sigma E\ ;\ J_D = \left(\frac{dD}{dt}\right),$$

   Where, J_C = conduction current density; J_D = Displacement current density; σ = Conductivity; E = Electric field Intensity and D = Electric flux Density.

5. State Biot-savart's Law.
   Ans 5. For answer refer: **Page 107, topic 3.4**, fig 3.6 & formula.

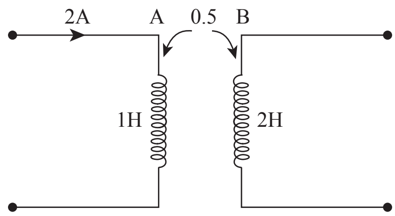
6. State Ampere's law.
   Ans 6. For answer refer: **Page 112, topic 3.6**.

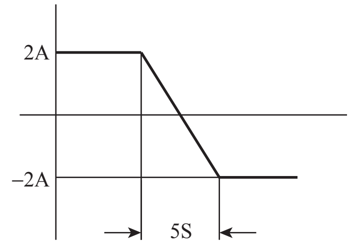
7. State the Faraday's law in differential form. How it is derived in integral form?
   Ans 7. For answer refer: **Page 165**,1st Paragraph and formula.

$$\oint E \cdot d\ell = -\mu \iint \left(\frac{\partial H}{\partial t}\right) \cdot ds;\ \nabla \times E = -\mu \left(\frac{\partial H}{\partial t}\right).$$

8. Two coils A and B having inductances of 1H and 2H respectively are placed coaxially. A current of 2A is reversed through coil A in 5 sec. If the Coefficient of mutual coupling is 0.5, find the EMF induced in coil B.

Ans 8.

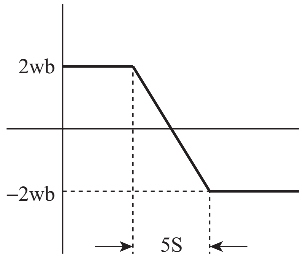




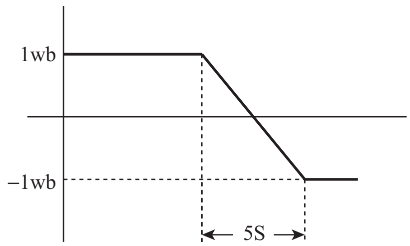
$$\phi = LI;\ \phi_A = 1(2) = 2Wb$$

Since coefficient of coupling is 0.5, flux linking the secondary $\phi_B = 0.5;\ \phi_A = 0.5(2) = 1\ Wb$

$$\Delta\phi = 1-(-1) = 2Wb.$$



Flux in coil A



Flux in coil B

Emf induced $= \dfrac{\Delta\phi}{\Delta t} = \dfrac{2Wb}{5\sec} = 0.4$ Volts.

9. Define the term "depth of penetration."
Ans 9. Depth of penetration or skin depth is the depth upto which the AC current flows from the surface. **Page 199, first 3 line**.

10. State two properties of plane waves.
Ans 10. For answer refer: **Page 172, topic 5.1.2**, First 2 point.

PART B – (5 × 11 = 55 marks)

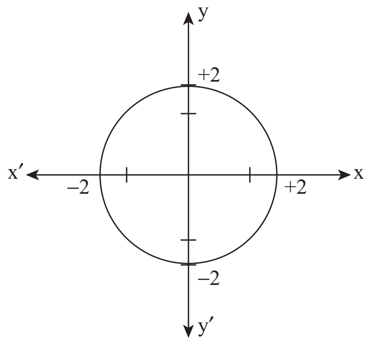
Answer ALL questions, One from each Unit. All questions carry equal marks.

UNIT I

11. (a) A circle of radius 2 units is centered at the origin. If $\overline{A} = 3y^2\overline{i} + 4z\overline{j} + 6y\overline{k}$, find $\oint \overline{A}.\overline{dl}$ along the path which coincides with circumference of the circle. (6)
Ans 11. (a) Refer Vector calculus topic in "Engineering Mathematics" Dr. M.K. Venkataraman. $\vec{A} = 3y^2\hat{i} + 4z\hat{j} + 6y\hat{k}$

Let the circle of 2 units lie in xy plane.

(ie) $x^2 + y^2 = 2^2 = 4.$



At Z = 0, (ie) at xy-plane, $\vec{A} = 3y^2\hat{i} + 6y\hat{k}$

"dℓ" for a circle is given by $d\ell = \hat{i}\,dx + \hat{j}\,dy$

$$\vec{A}\cdot d\ell = \left(3y^2\hat{i} + 6y\hat{k}\right)\cdot\left(\hat{i}\,dx + \hat{j}\,dy\right)$$

$$\oint_\ell \vec{A}\cdot d\ell = \oint_\ell 3y^2\,dx$$

To evaluate the integral let's take $x = \cos\theta$; $y = \sin\theta$; $dx = -\sin\theta d\theta$. θ needs to be varied from 0 to 2⇔.

$$\oint 3y^2 dx = \int_0^{2\pi} 3(\sin\theta)^2(-\sin\theta)\,d\theta = -3\int_0^{2\pi}(\sin\theta)^3\,d\theta$$

$$= -3\int_0^{2\pi}\sin\theta(\sin\theta)^2\,d\theta = -3\int_0^{2\pi}\sin\theta\left[1-(\cos\theta)^2\right]d\theta$$

$$= -3\left[\int_0^{2\pi}\sin\theta\,d\theta - \int_0^{2\pi}\sin\theta(\cos\theta)^2\,d\theta\right]$$

$$= -3\left[\left[-\cos\theta\right]_0^{2\pi} - \int_0^{2\pi}\sin\theta(\cos\theta)^2\,d\theta\right] \qquad \text{(I)}$$

$$\int \sin\theta(\cos\theta)^2\,d\theta.$$

Let $u = \cos\theta$; $du = -\sin\theta\,d\theta$

(ie) $$\int u^2(-du) = -\int u^2\,du = -\left(\frac{u^3}{3}\right) = -\frac{(\cos\theta)^3}{3}$$

So Eq (I) becomes,

$$= -3\left[\left[-\cos\theta\right]_0^{2\pi} + \frac{1}{3}\left[(\cos\theta)^3\right]_0^{2\pi}\right]$$

$$= -3\left[\left(-\cos 2\pi + \cos 0\right) + \frac{1}{3}\left\{(\cos 2\pi)^3 - (\cos 0)^3\right\}\right]$$

$$= -3\left[(-1+1) + \frac{1}{3}\left\{1^3 - 1^3\right\}\right] = 0.$$

11. (b) Find the divergence of the vector function. $\overline{A} = x^2\overline{i} + x^2y^2\overline{j} + 24x^2y^2z^3\overline{k}$ Evaluate the integral of this divergence throughout the volume of a unit cube centered at the origin. (5)

Ans 11. (b) Refer Vector calculus topic in "Engineering Mathematics" volume Two by Dr. M. K. Venkataraman.

$$\vec{A} = x^2\hat{i} + x^2y^2\hat{j} + 24x^2y^2z^3\hat{k}$$

$$\nabla = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial y}\right)$$

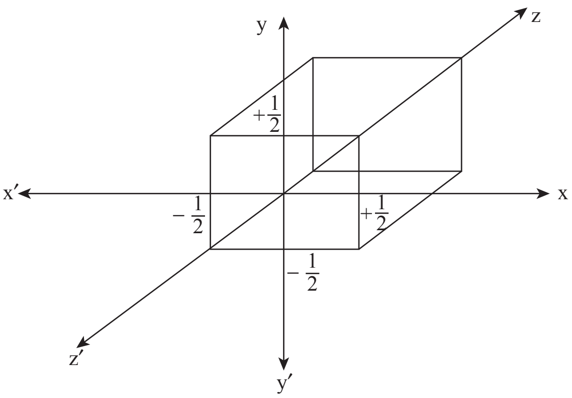
$$\nabla \cdot A = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(x^2\hat{i} + x^2y^2\hat{j} + 24x^2y^2z^3\hat{k}\right)$$

$$= \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(x^2y^2) + \frac{\partial}{\partial z}(24x^2y^2z^3).$$

$$= 2x + 2x^2y + 24x^2y^2(3z^2)$$

$$\nabla \cdot A = 2x + 2x^2y + 72x^2y^2z^2.$$

$$\oint_v (\nabla\cdot A)\,dv = \int_{-\frac{1}{2}}^{+\frac{1}{2}}\int_{-\frac{1}{2}}^{+\frac{1}{2}}\int_{-\frac{1}{2}}^{+\frac{1}{2}} (\nabla\cdot A)\,dx\,dy\,dz$$



$$\oint_v (\nabla\cdot A)\,dv = \int_{-\frac{1}{2}}^{+\frac{1}{2}}\int_{-\frac{1}{2}}^{+\frac{1}{2}}\int_{-\frac{1}{2}}^{+\frac{1}{2}} \left(2x + 2x^2y + 72x^2y^2z^2\right)dx\,dy\,dz$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(2x\,dx + 2yx^2\,dx + 72y^2z^2x^2\,dx\right) dy\,dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[2\left(\frac{x^2}{2}\right) + 2y\left(\frac{x^3}{3}\right) + 72y^2z^2\left(\frac{x^3}{3}\right)\right]_{-\frac{1}{2}}^{+\frac{1}{2}} dy\;dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[x^2 + \left(\frac{2}{3}\right) y\,x^3 + 24y^2z^2x^3\right]_{-\frac{1}{2}}^{+\frac{1}{2}} dy\;dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[\left(\frac{1}{4} - \frac{1}{4}\right) + \frac{2}{3}y\left(\frac{1}{8} + \frac{1}{8}\right) + 24y^2z^2\left(\frac{1}{8} + \frac{1}{8}\right)\right] dy\;dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[0 + \frac{\cancel{2}}{3}y\left(\frac{\cancel{2}}{\underset{2}{\cancel{8}}}\right) + \overset{3}{\cancel{24}}\;y^2\;z^2\left(\frac{2}{\cancel{8}}\right)\right] dy\;dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[\frac{1}{6}(y) + 6y^2z^2\right] dy\;dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[\left(\frac{1}{6}\right) y\,dy + 6y^2z^2\;dy\right] dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[\left(\frac{1}{6}\right)\left(\frac{y^2}{2}\right) + 6z^2\left(\frac{y^3}{3}\right)\right]_{-\frac{1}{2}}^{+\frac{1}{2}} dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[\frac{1}{12}(y^2) + 2z^2(y^3)\right]_{-\frac{1}{2}}^{+\frac{1}{2}} dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[\frac{1}{12}\left(\frac{1}{4} - \frac{1}{4}\right) + 2z^2\left(\frac{1}{8} + \frac{1}{8}\right)\right] dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[0 + 2z^2\left(\frac{2}{8}\right)\right] dz$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left(\frac{1}{2}\right) z^2\, dz = \frac{1}{2}\left[\frac{z^3}{3}\right]_{-\frac{1}{2}}^{+\frac{1}{2}}$$

$$= \frac{1}{6}\left[z^3\right]_{-\frac{1}{2}}^{+\frac{1}{2}} = \frac{1}{6}\left[\frac{1}{8} + \frac{1}{8}\right] = \frac{1}{6}\left(\frac{2}{8}\right) = \frac{1}{24}$$

(ie) $\int_V (\nabla \cdot A)\, dV = \left(\frac{1}{24}\right).$

12. (a) Transform the vector given below into cylindrical coordinates.

$\overline{F} = z\bar{i} + (1-x)\,\overline{j} + \frac{y}{x}\overline{k}$ (5)

Ans 12. (a) Similar problem solved at page 13, Q 1.1

$$\vec{F} = z\hat{i} + (1-x)\,\hat{j} + \left(\frac{y}{x}\right)\hat{k}$$

So transform from Cartesian to cylindrical coordinates, we have,

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z \\ (1-x) \\ \left(\frac{y}{x}\right) \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi\, z + (1-x)\sin\phi + 0 \\ -\sin\phi\, z + (1-x)\cos\phi + 0 \\ 0 + 0 + \frac{y}{x} \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} z\cdot\cos\phi + (1-x)\sin\phi \\ -z\cdot\sin\phi + (1-x)\cos\phi \\ \left(\frac{y}{x}\right) \end{bmatrix}$$

$$A = [z\cos\phi + (1-x)\sin\phi]A_\rho + [(1-x)\cos\phi - z\sin\theta]A_\phi + \left(\frac{y}{x}\right)A_z.$$

12. (b) State Gauss's law and use it to determine the field intensity due to an infinitely long line charge. (6)

Ans 12. (b) For answer refer: **Page 37, topic 1.9 and topic 1.9.3** in page 39.

UNIT II

13. (a) What is an electric dipole? Obtain an expression for potential and electric field intensity due to a dipole at a distant point P. (6)

Ans 13. (a) For answer refer: **Page 64, topic 2.5.1, topic 2.5.2** in page 66.

13. (b) Derive the expression for the total energy in an electrostatic field. (5)

Ans 13. (b) For answer refer: **Topic 2.10, in page 78**.

Or

14. (a) The Polarization within a region having ε_r = 2.26 has the uniform value $\overline{P} = -2\overline{i} + 7\overline{j} + 3\overline{k}$ nC/m^2. find

(i) $\overline{E}$

(ii) $\overline{D}$, and

(iii) the magnitude of voltage gradient. (6)

Ans 14. (a) For answer refer: **Page 64,** formula in the first half of the page.

$$D = \varepsilon_0(1+\chi_e)E.$$
$$\varepsilon_r = (1+\chi_e)$$
$$P = \chi_e \;\; \varepsilon_0 \;\; E$$

$$\chi_e = \varepsilon_r - 1 = 2.26 - 1 = 1.26$$

$$\vec{E} = \left(\frac{1}{\chi_e\varepsilon_0}\right)\cdot\vec{P} = \left(\frac{1}{1.26\times\varepsilon_0}\right)\left(-2\hat{i} + 7\hat{j} + 3\hat{k}\right)\times 10^{-9} \text{ C/m}^2$$

$$= 896.35\left[-2\hat{i} + 7\hat{j} + 3\hat{k}\right] \text{V/m}$$
$$= -1792.7\hat{i} + 6274.5\hat{j} + 2689\hat{k}$$

(i) $\vec{E} = -1792.7\hat{i} + 6274.5\hat{j} + 2689\hat{k}$

(ii) $\vec{D} = \varepsilon_0(1+\chi_e)E = \varepsilon_0\ \varepsilon_r\ E$

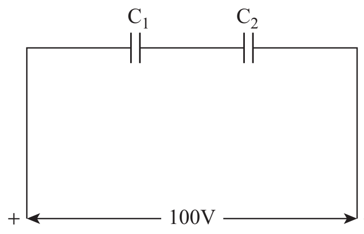
$$= 2.00 \times 10^{-11}\left[-1792.7\hat{i} + 6274.5\hat{j} + 2689\hat{k}\right]$$

$$= \left[-35.88\hat{i} + 125.55\hat{j} + 53.80\hat{k}\right] \times 10^{-9}$$

(iii) Voltage gradient $= \left(\dfrac{dV}{d\ell}\right) = E$

Magnitude of voltage gradient $= |E| = \sqrt{(-1792.7)^2 + (6274.5)^2 + (2689)^2} = 7057.9$ V/m.

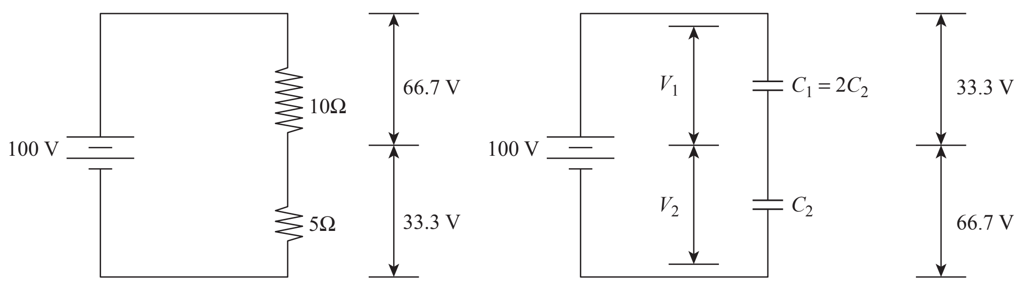
14. (b) Find the voltage across C_1 and C_2 in the figure show below: it is given that $C_1 = 2C_2$.



(5)

Ans 14. (b) Resistor and Capacitor possess opposite functionality.

More the resistance, more the voltage drop across it in series connection.
In case of capacitor, it is opposite; more the capacitance, less the voltage drop across it.



In a capacitor, $Q = CV$. Here for series capacitors, the charge on both is same, hence,

$$Q = C_1V_1 \qquad (1)$$

$$Q = C_2V_2 \qquad (2)$$

$$\frac{(1)}{(2)} : \frac{Q}{Q} = \frac{C_1V_1}{C_2V_2} \qquad \Rightarrow \frac{C_1V_1}{C_2V_2} = 1$$

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}; \qquad \Rightarrow \frac{V_1}{V_2} = \frac{C_2}{2C_2} = \left(\frac{1}{2}\right)$$

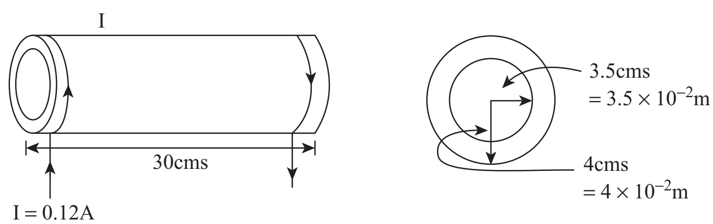
$$V_2 = 2V_1$$
$$V_1 + V_2 = 100 \text{ V}$$

$$V_1 + 2V_1 = 100V \qquad \Rightarrow 3V_1 = 100 \text{ V}; \qquad V_1 = \left(\frac{100}{3}\right)\text{V}$$

$$V_1 = 33.33 \text{ V}; \qquad V_2 = 2\cdot V_1 = 66.67 \text{ V}.$$

UNIT III

15. Show that the field strength at the end of a long solenoid is one half of that at the center. A solenoid of length 30 cm is wound on a wooden core, the external and internal diameters of which are 4.0 and 3.5 cm respectively. If the current in the coil is 0.12 A, find the number of turns required to produce a magnetizing force of 750 AT/m at the center of the coil. (11)
Ans 15. For answer refer: **Page 122, topic 3.7.3** fully.



Magnetizing force H at the centre of the coil,

$$H = \frac{B}{\mu_0} = \left(\frac{N}{2}\right)\frac{I}{\sqrt{a^2 + \left(\frac{L}{2}\right)^2}}$$

(ie) $750 \text{ AT/m} = \left(\frac{N}{2}\right)\frac{0.12}{\sqrt{(3.75\times10^{-2})^2 + (15\times10^{-2})^2}}$

$$750 = 388.06\times10^{-3} \text{ N}$$

$$N = \frac{750}{388.06\times10^{-3}} = 1933$$

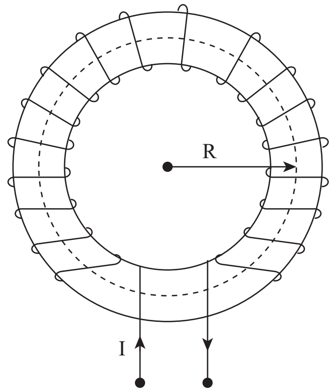
No. of Turns, N = 1933.

Or

16. (a) Discuss the application of Biot-Savart's law to circuit current loop and obtain an expression for H at the center of circular current carrying loop. (6)
Ans 16. (a) For answer refer: **Page 119, topic 3.7.2** fully, till "when h = 0....".

16. (b) An iron ring 0.2 m in diameter and 10 m² cross sectional area of the core is uniformly wound with 250 turns of wire. If the density in the core is 1 Tesla, permeability of iron is 500, find the exciting current in the winding. Determine also self inductance and stored energy.

Ans 16. (b) Diameter = 0.2 m[Refer **Topic 3.7.4 in page 123**]
Radius R = 0.1 m = 1×10^{-1}m
Cross section area of core = 10 cm^2 = $10 \times (10^{-2}\ m)^2 = 10\times 10^{-4}\ m^2$
N = 250 turns
B = 1 Tesla
μ_r = 500



$$H = \frac{NI}{2\pi R}; \qquad B = \mu H$$

$$B = \mu_0\ \mu_r \left(\frac{NI}{2\pi R}\right)$$

$$1 = \frac{\mu_0 (500)\cdot(250)\cdot I}{2\pi(1\times10^{-1})}$$

$$1 = 0.25\ I;\ \ I = \left(\frac{1}{0.25}\right) = 4A$$

Refer **Topic 3.13.2, in page 134**.

$$L = \frac{\mu N^2 r^2}{2R} = \frac{\mu N^2(\pi r^2)}{2R\pi} = \frac{\mu_0 (500)\,(250)^2 10\times10^{-4}}{2\times1\times10^{-1}\times\pi}$$

$$L = 62.5\times10^{-3}\ \text{H}$$

Energy stored E $= \frac{1}{2}LI^2 = \frac{1}{2}(62.5\times10^{-3})\cdot(4)^2$

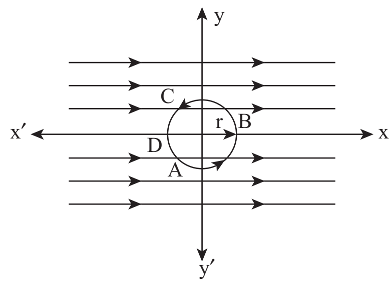
= 0.5 W.

UNIT IV

17. (a) Derive poynting theorem and give its significance.
Ans 17. (a) For answer refer: **Page 204**, poynting theorem fully.

17. (b) A differential current loop is in $z = 0$ plane and is placed in a magnetic flux density B wb/m^2. Find the total torque on the loop about the origin at the center of the loop.

Ans 17. (b) Let the area of the differential current loop be "A" and the current passing thro' it is "I". As it is placed in a magnetic flux density of "B" Wb/m^2, the force experienced by it is $F = BI\ell \sin\theta = BI(2\pi r)\cdot \sin\theta$



For maximum force, the flux and the current in conductor must be perpendicular to one another. This happens at B and D.

At "A" and "C", they are parallel to the flux lines and the force becomes zero.

θ starts at "A", θ reaches "B" at 90°, Reaches "C" at 180°, and D at 270°.

For given configuration of flux and current, At B, force F acts downwards; while at D, the force acts upwards.

Hence, Torque T = 2F × r

Torque is given by $T = 2F \times r = 2BI(2\pi r)\sin\theta \cdot r$

$$T = 2BI\,2\pi r^2 \cdot \sin\theta = 4BI(\pi r^2)\cdot \sin\theta$$

$$T(\theta) = 4BIA \cdot \sin\theta.$$

Or

18. Starting from Maxwell's equations derive wave equation for a time harmonic electromagnetic field in a conducting medium as copper and obtain the expression for the electric and magnetic field intensities in terms of constants of the medium and the frequency.

Ans 18. For answer refer: **Page 176, topic 5.3, 5.3.1 and 5.3.3**.

UNIT V

19. (a) State and explain the distinction between horizontal polarization and vertical polarization.

Ans 19. (a) For answer refer: **Page 176, Topic 5.2.2**.

19. (b) Discuss reflection at the surface conducting medium.

Ans 19. (b) For answer refer: **Page 206, topic 5.9** till page 210 "1.Special case …… conclusion: the wave is totally reflected."

Or

20. A plane electromagnetic wave travelling in the positive z direction in an unbounded lossless dielectric medium with μ_r = 1 and ε_r = 3 has peak electric field intensity E of 16 V/m. find

(i) V (ii) η (iii) $|\overline{H}|$ (iv) $|\overline{P}|$ (8)

Ans 20. (a) $\mu_r = 1;\ \varepsilon_r = 3;\ E_m = 16$ V/m

(i) Velocity, $v = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{1}{\sqrt{\mu_0\ \mu_r\ \varepsilon_0\ \varepsilon_r}} = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}\ \sqrt{\mu_r\varepsilon_r}}$

Velocity $v = \dfrac{3\times10^8}{\sqrt{\mu_r\ \varepsilon_r}} = \dfrac{3\times10^8}{\sqrt{1\times3}} = \sqrt{3}\times10^8$ m/sec

(ii) Intrinsic Impedance, $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}}$ for a lossless dielectric, $\sigma = 0$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\left(\frac{\mu}{\varepsilon}\right)} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{\mu_0(1)}{\varepsilon_0(3)}}$$

$$\eta = 217.50$$

(iii) $|\vec{H}| = \left|\dfrac{\vec{E}_m}{\eta}\right| = \left(\dfrac{16}{217.50}\right) = 73.56\times10^{-3}$

(iv) $|\vec{P}| = \left|\dfrac{E_m^2}{120\pi}\right| = \dfrac{16^2}{120\pi} = 679.06$ Watts.

20. (b) Write the significance of intrinsic impedance. (3)

Ans 20. (b) For answer refer: **Page 210**, special case (I)

Based on Intrinsic impedance the reflection and transmission coefficient of EM wave can be found. From this the percentage of EM wave transmitted and reflected can be found.